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Technical Note

No. 265

A  $\gamma_t$ -Jump Scheme for the Brookhaven AGS

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I. Introduction

AGS beam losses at transition are now tolerable ( $<5\%$ ), but as the present improvement plans are implemented and the intensity is increased, new mechanisms will become important and the losses will increase. This Note describes studies directed towards minimizing these losses.

Werner Hardt has studied these losses at the CERN PS<sup>1</sup>. In particular, he found that by sharply reducing the time spent going through transition he could reduce the losses. Hardt visited Brookhaven in early June, 1986, in order to help us better understand the AGS. As a result of work inspired by his visit, we now believe that intensities of  $\sim 5 \times 10^{13}$  circulating protons are attainable in the AGS without significant losses at transition.

II. A Qualitative Look at Transition Losses

The angular velocity,  $\omega = v/R$ , of an orbiting particle in a synchrotron can either increase or decrease with energy. As a particle gains energy,  $E$ , its velocity,  $v$ , increases -- though for highly relativistic particles  $\Delta v/v \ll \Delta E/E$ . On the other hand,  $\Delta R/R = \alpha \Delta P/P \approx \alpha \Delta E/E$  as the particle becomes highly relativistic, with  $\alpha$  independent of energy. The point at which the change in  $R$  is greater than the change in  $v$  and the angular velocity begins to decrease with energy is called "transition". The energy at which this occurs is called the "transition energy", usually denoted as  $\gamma_t$ , and depends on the focusing properties of the machine.  $\gamma_t$  is usually about equal to the horizontal tune,  $Q_H$ .

It is convenient to view transition in terms of "mass". Below this point, as a particle is accelerated its angular velocity increases: the harder it's pushed, the faster it goes. Above transition, the situation is just the opposite: pushing decelerates; a particle behaves as if it had a negative mass. Below transition when the mass is positive, the phase of the rf acceleration voltage is adjusted so a stable particle rides the leading edge of the wave (Fig. 1a):

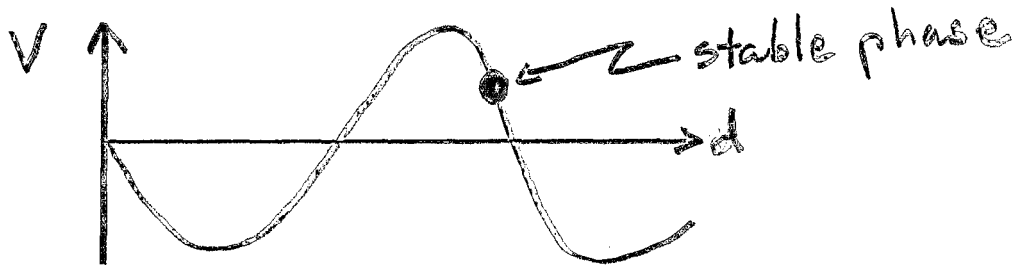


Figure 1a

A slower particle sees a higher voltage and will speed up, a faster particle a lower voltage. The situation after transition is shown in Fig. 1b:

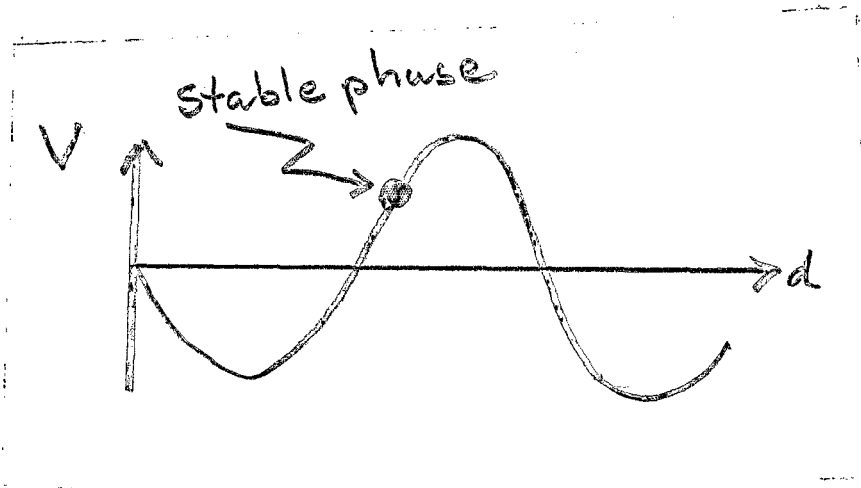
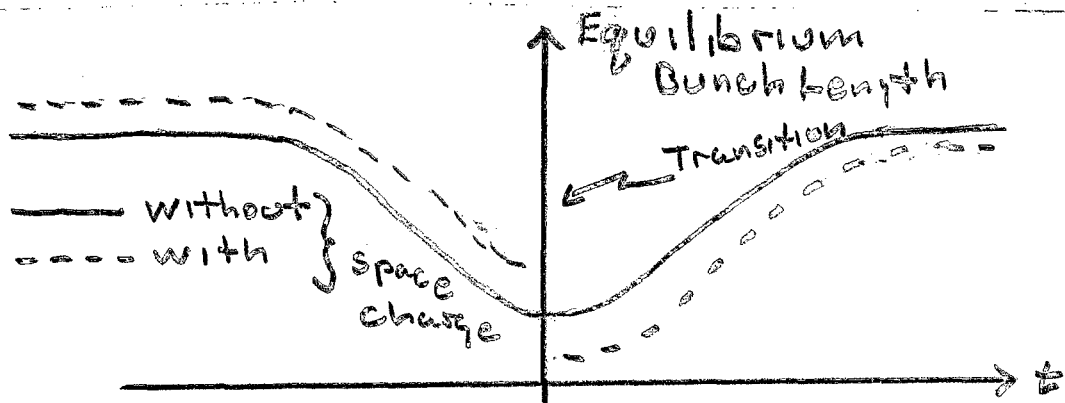


Figure 1b.

In order to slow down a faster particle with negative mass, it must experience a higher voltage; a slower particle is sped up by a lower voltage. To accomplish this, the rf phase is changed, as shown in the figures. But the change in behavior is not quite so discontinuous. As a particle nears transition, its mass is positive and increasing. At transition its mass appears infinite. Above transition, the magnitude of the "negative" mass begins to decrease.

Consider, now, a bunch of particles undergoing acceleration. Below transition, the particles will experience (synchrotron) oscillations about an equilibrium point with (as the mass increases) a constantly decreasing bunch length and frequency. Above transition, the bunch length and frequency will both increase. However, since the electrostatic (space charge) forces between the particles are always repulsive, their effects will be opposite according to whether the mass is "positive" or "negative". In the "positive" region below transition, the repulsive space charge forces will spread the particles out and increase the equilibrium bunch length. In the "negative" region above transition, this repulsive force will attract the particles and decrease the equilibrium bunch length. (This is somewhat like the situation in Saturn's rings, where the attractive space charge (gravity) spreads the dust particles apart.) This behavior is illustrated in Fig. 2.



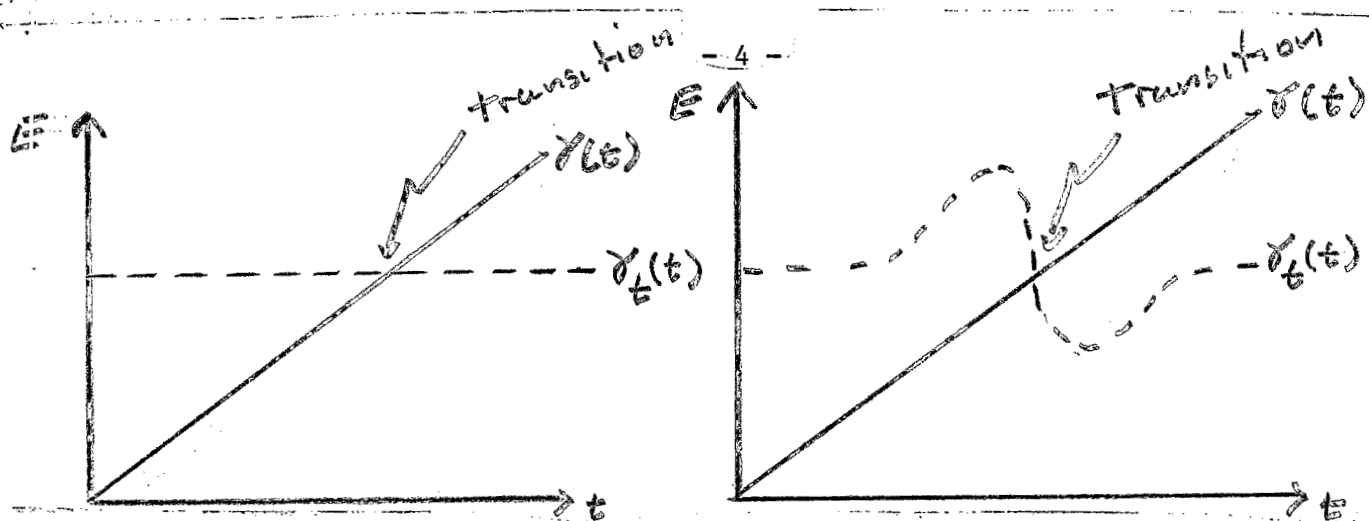
Note particularly the discontinuity in equilibrium bunch length at transition. Because of this discontinuity, oscillations can be excited which result in particle loss. In general, the size of these oscillations depends on the mis-match between the equilibrium bunch length before and after transition, which in turn depends on the beam intensity.

In any situation with "negative mass" undamped oscillations can develop. At transition, the frequency spread of the synchrotron oscillations is small as is the resulting Landau damping. Thus, just above transition a situation exists in which growing oscillations can be excited; this is the "negative mass instability". The size of these oscillations (and the resulting emittance blow-up) depends on the beam intensity and the time spent in the regime with little damping.

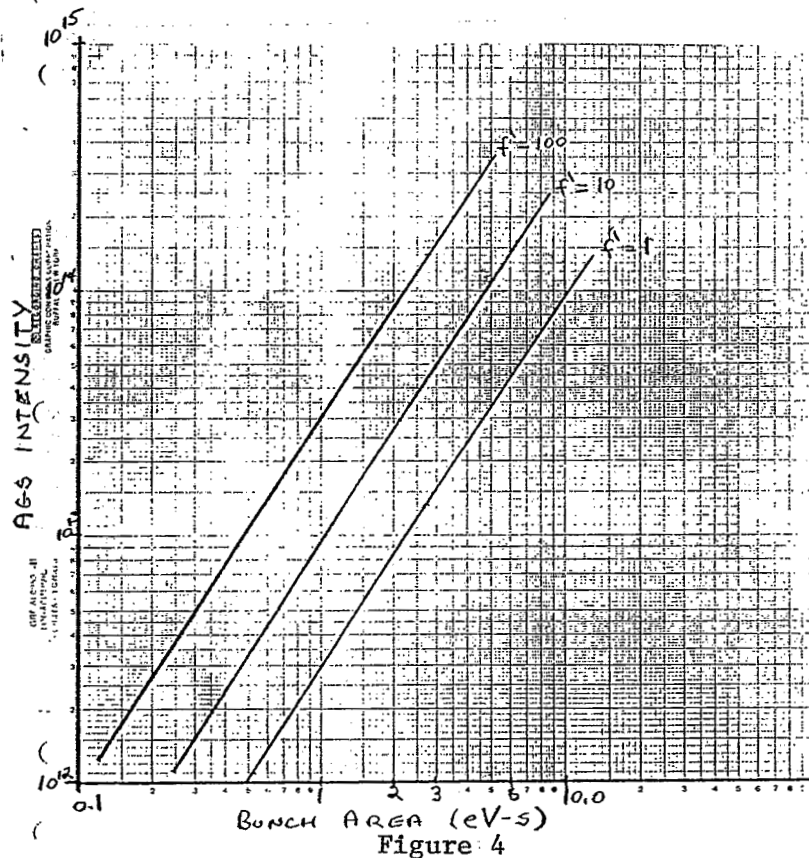
Two approaches can be taken to reduce these transition losses: artificial enlargement of the bunches before transition so as to reduce space charge forces; and minimizing the time spent in the unstable region. The balance of this Note is principally concerned with the second approach.

### III. $\gamma_t$ - Jump

Hardt's idea, which has been implemented at CERN, was based on the observation that quadrupole pairs separated by  $1/2$  betatron wavelength and configured as doublets can alter  $\gamma_t$  of a synchrotron without affecting its tune. By pulsing such quadrupoles, the time spent near transition can be reduced. This is illustrated in Figs. 3a and 3b:



In the the lefthand figure,  $\gamma_t$  remains a fixed parameter of the accelerator lattice. In the righthand figure, the quadrupoles are pulsed so that  $\gamma_t$  is a rapidly changing function of time at transition. Crossing speed enhancements of the order of 30-50 are attainable with this technique. Hardt<sup>2</sup> has established criteria for lossless transition and has parameterized his results in a convenient form for the AGS (Fig. 4):



Here, intensity for lossless transition crossing is plotted as a function of bunch area and intensity. Lines of constant crossing speed enhancements,  $f'$  (as defined in Reference 1), are shown. The best, though not usual, AGS operation has been at  $f' = 4$ .

#### IV. Computer Simulations

Using the general accelerator design program MAD<sup>3</sup>, we have investigated several sets of quadrupole configurations. The initial criterion used in setting these simulations was to try realizable magnets and deployments (i.e., existing or easily-constructed quadrupoles in real AGS straight sections) which most closely fulfilled Hardt's requirement: 1/2 betatron wavelength separation for the magnets comprising a doublet. We soon realized that 3/2 betatron wavelength separation also could change  $\gamma_t$  without affecting  $Q_H$  and expanded our studies to include such configurations. We present here our most encouraging result, while a complete summary of these studies appears as the Appendix.

The most successful configuration used six existing "slow" quadrupoles configured as three doublets with 3/2 betatron wavelength separation. (This configuration is denoted "W<sup>+</sup>" in the Appendix.):

$$\frac{B17 + D17-}{\text{doublet}} \quad \frac{F17 + H17-}{\text{doublet}} \quad \frac{J17 + L17-}{\text{doublet}}$$

where the locations and polarities are indicated. The result of this simulation is presented in the following table:

| Quad Strength (K)        | 0.0*  | 0.2   | 0.25   | 0.30   | 0.35   |
|--------------------------|-------|-------|--------|--------|--------|
| $Q_H$                    | 8.711 | 8.681 | 8.665  | 8.647  | 8.625  |
| $Q_V$                    | 8.800 | 8.796 | 8.793  | 8.790  | 8.787  |
| $(\beta_x)_{\text{max}}$ | 22.5  | 35.8  | 40.0   | 44.4   | 49.0   |
| $(\beta_y)_{\text{max}}$ | 22.3  | 27.3  | 28.7   | 30.0   | 31.4   |
| $(dx)_{\text{max}}$      | 2.16  | 7.78  | 8.98   | 9.99   | 10.82  |
| $\gamma_t$               | 8.449 | 9.667 | 10.366 | 11.247 | 12.336 |
| $\Delta\gamma_t$         | 0     | 1.217 | 1.916  | 2.797  | 3.886  |

\*Unperturbed machine, as calculated by MAD.

We see that substantial changes in  $\gamma_t$  are possible without producing unacceptable changes in other machine parameters.

#### V. Attainable Intensity

In order to evaluate the potential improvement in AGS intensity, we must now consider how fast the quads can be pulsed. Using the existing fast modulators (which were installed as part of the polarized proton program), we have determined that the magnets can reach 425 Amperes (which corresponds to  $K=0.35$ ) in 2.2 msec. This yields a  $\Delta\gamma/\Delta t$  of 1770/sec or, since  $(\Delta\gamma/\Delta t)_0 = 60$ , an  $f'$  of 30. We can now redraw the "Hardt Plot", Fig. 4, with the line  $f'=30$  (Fig. 5).

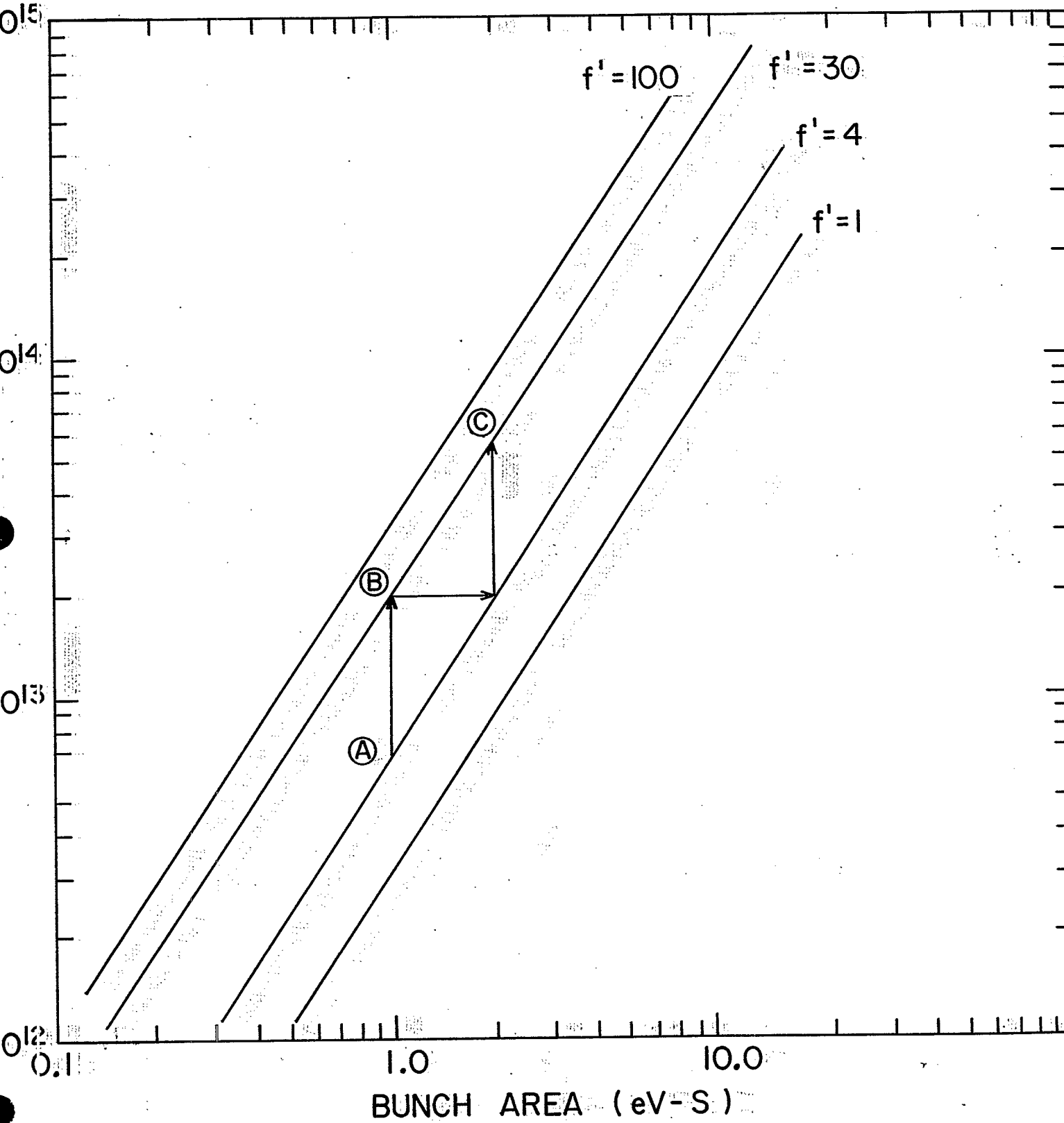


Figure 5

Point "A" corresponds to the highest intensity possible in the present AGS with clean passage through transition:  $f'=4$ , bunch area 1 eV-second. By going to the line  $f'=30$ , we move to point "B"-- an intensity improvement of about a factor of three. If rf blow-up is used to double the bunch area before transition<sup>4</sup>, we can operate it at point "C". Thus, it appears that lossless transition at an intensity of  $4-5 \times 10^{13}$  is possible.

## VI. Future Plans, Conclusion

During a brief studies period in July, 1986, we were able to pulse some existing quadrupoles and to verify the accuracy of MAD predictions<sup>5</sup>. Cabling is presently being installed which will permit us to perform tests designed to verify the predictions of the previous section. These tests are scheduled for the fall of 1986. Should they prove successful, a  $\gamma_t$ -jump will have been implemented at the AGS.

## REFERENCES

1. Hardt, W. "Gamma Transition-Jump Scheme of the CPS". Proceedings of the 9th International Conference on High Energy Particle Accelerators, Stanford, 1974.
2. Hardt, W. Unpublished.
3. Isline, F.C. "MAD (Methodical Accelerator Design)", CERN LEP TH/85-15.
4. Goldman, M., et al. To be published.
5. Ratner, L. AGS Studies Report No. 208 (1986).

## APPENDIX

Proposals for one- and two-family sets of quadrupole lenses to alter  $\gamma_t$  in the AGS have been evaluated using the MAD code to calculate tunes, betas, dispersions and transition-gammas for various configurations of the quads.

The data for each configuration is in a separate table. Criteria for acceptable proposals are:

- (1) changes in tune are minimal (less than .05)
- (2) beta-max is not increased by more than a factor of the order of 4.
- (3) dispersion is not increased by more than a similar factor
- (4) a change in transition-gamma of the order of 1.0 or more be achieved.

Several two-family configurations, suggested by Werner Hardt, involve 8 lenses per half-machine -- 16 lenses in all. These configurations are numbered 6 through 10.

Configuration #6 produces moderate horizontal tune-shifts for the weaker strengths ( $K=.15/.20$ ); for the stronger strengths, the tune-shifts are not acceptable. The weaker strengths produce a  $\Delta\gamma_t$  of 1.36 with only modest increases in beta-max and dispersion. Configuration #7 has similar characteristics and a  $\Delta\gamma_t$  of 1.39 for the weaker strengths. Configurations #8 and #9 are unstable in y for the stronger strengths. For the weaker ones, the y tune-shifts are unacceptable.  $\Delta\gamma_t$ 's are 1.45 and 1.40. Configuration #10 is a one-family, four-superperiod set (again 16 lenses in all). Tune-shifts and beta-max's are well within acceptable limits. Maximum dispersion may be a problem for stronger strengths ( $K=.30$  and possibly  $K=.25$ ).  $\Delta\gamma_t = 1.335$  and  $1.847$  for  $K = .20$  and  $.25$ , respectively.

The best candidates from these sets are Configurations #6 and 7 with weaker strengths. If a somewhat larger dispersion (of the order of .10) is tolerable, then the one-family Configuration #10 at  $K = .25$  is better.

Larry Ratner proposed configurations using magnets already in place at "15" locations in various sectors: the resulting spacing is approximately  $3/2$ -wave-length; these gave poor results. Alternatively, he suggested using the "5" locations. These configurations are noted "Y", "X", "W" and "V", and require only 6 or 8 lenses.

Configuration Y(+) has good beta-max and maximum dispersion for  $K=.15$  and  $.20$ ; for  $K=.25$  dispersion may be too large.  $\Delta\gamma_t = 0.77$  for  $K=.20$ . Configuration X(-) (2) at  $K=.20$  gives a  $\Delta\gamma_t = 1.04$ ; all other parameters for this configuration are good. Configuration W(+) at  $K=.20$  and  $.25$  have excellent beta-max values and a modest tune shift in x; maximum dispersion is acceptable, though high;  $\Delta\gamma_t = 1.21$  and  $1.91$  respectively. Configuration V has no good candidates.

A K-value between 0.20 and 0.25 in configuration W(+) is the best choice among these proposals.

Another  $3/2$ -wave-length set using 8 lenses was proposed by Elliot Auerbach. These involve a 42-magnet spacing to approximate  $3/2$ -wave-length (the more precise value is 41.3). They are labelled "E1" and "E3".

Configuration E1, operated as a two-family set, gives  $\Delta\gamma_t = 0.95$  for  $K=.20$ ; beta-max, maximum dispersion and x tune-shifts are small; the y tune-shift is about .04. If K is increased to .25,  $\Delta\gamma_t = 1.52$ , but maximum dispersion goes up to 9 meters and the y tune-shift goes to .06. Configuration E3, the best of this group, gives  $\Delta\gamma_t = 1.55$  at  $K=.20$  with good beta-max and moderately high, though tolerable, maximum dispersion.



## CONFIGURATION #6:

Run No. 6/1 6/2 6/3 6/4

K-slow .20 .20 .30 .30

K-fast .15 .15 .20 .20

Rel. Polarity + - + -

Q-x 8.743 8.762 8.782 8.826

Q-y 8.788 8.793 8.777 8.785

Beta-max (x) 44 61 63 112

Beta-max (y) 40 38 52 49

(dx)-max 6.17 4.59 8.12 6.16

Gamma-tr 7.818 9.174 7.364 10.178

 $\Delta \gamma_f$ | | | |  
---1.36--- ---2.81---

| SLOW |   | FAST |   |
|------|---|------|---|
| A14  | + | D14  | + |
| B8   | - | E8   | - |
| B18  | + | E18  | + |
| C12  | - | F12  | - |

(repeated in  
Sectors G-L)

## CONFIGURATION #7:

Run No. 7/1 7/2 7/3 7/4

K-slow .20 .20 .30 .30

K-fast .15 .15 .20 .20

Rel. Polarity + - + -

Q-x 8.749 8.763 8.790 8.8286

Q-y 8.794 8.794 8.786 8.786

Beta-max (x) 46 59 67 107

Beta-max (y) 40 39 53 52

(dx)-max 6.42 4.52 8.61 6.06

Gamma-tr 7.790 9.184 7.289 10.224

 $\Delta \gamma_f$ | | | |  
---1.39--- ---2.93---

| SLOW |   | FAST |   |
|------|---|------|---|
| A18  | + | D18  | + |
| B12  | - | E12  | - |
| C2   | + | F2   | + |
| C16  | - | F16  | - |

(repeated in  
Sectors G-L)

## CONFIGURATION #8:

|                  |                |       |          |       |
|------------------|----------------|-------|----------|-------|
| Run No.          | 8/1            | 8/2   | 8/3      | 8/4   |
| K-slow           | .20            | .20   | .30      | .30   |
| K-fast           | .15            | .15   | .20      | .20   |
| Rel. Polarity    | +              | -     | -        | +     |
| Q-x              | 8.693          | 8.699 | 8.685    | 8.676 |
| Q-y              | 8.841          | 8.886 | unstable | 8.894 |
| Beta-max (x)     | 40             | 37    | 47       | 53    |
| Beta-max (y)     | 46             | 97    | xxx      | 77    |
| (dx)-max         | 6.54           | 4.79  | 5.96     | 9.40  |
| Gamma-tr         | 7.771          | 9.217 | 10.193   | 7.195 |
| $\Delta\delta_t$ | <br>---1.45--- |       | XXX      |       |

| SLOW |   | FAST |   |
|------|---|------|---|
| A8   | + | D8   | + |
| B2   | - | E2   | - |
| B12  | + | E12  | + |
| C6   | - | F6   | - |

or

(repeated in Sectors G-L)

## CONFIGURATION #9:

|                  |                |       |          |       |
|------------------|----------------|-------|----------|-------|
| Run No.          | 9/1            | 9/2   | 9/3      | 9/4   |
| K-slow           | .20            | .20   | .30      | .30   |
| K-fast           | .15            | .15   | .20      | .20   |
| Rel. Polarity    | +              | -     | -        | +     |
| Q-x              | 8.699          | 8.700 | 8.687    | 8.684 |
| Q-y              | 8.846          | 8.887 | unstable | 8.902 |
| Beta-max (x)     | 40             | 37    | 51       | 54    |
| Beta-max (y)     | 48             | 95    | xx       | 83    |
| (dx)-max         | 6.29           | 4.81  | 6.00     | 8.85  |
| Gamma-tr         | 7.807          | 9.209 | 10.150   | 7.283 |
| $\Delta\delta_t$ | <br>---1.40--- |       | XXX      |       |

| SLOW |   | FAST |   |
|------|---|------|---|
| A4   | + | D4   | + |
| A18  | - | D18  | - |
| B8   | + | E8   | + |
| C2   | - | F2   | - |

or

(repeated in Sectors G-L)

## CONFIGURATION #10:

Run No.: 10/1 10/2 10/3

K-slow: .20 .25 .30

Q-x: 8.716 8.719 8.722

Q-y: 8.813 8.820 8.828

Beta-max(x): 41 47 53

Beta-max(y): 39 45 51

(dx)-max: 7.86 9.86 12.06

Gamma-tr: 7.114 6.602 6.112

 $\Delta x$ 

t

1.335 1.847 2.337

SLOW

A8 +

B2 -

B18 +

C12 -

(repeated  
four times)

CONFIGURATION Y: 8 lenses: (A-C, D-F, G-I, J-L) : 5

| Run No.                   | Y+    | Y-      | Y+    | Y-      | Y+    | Y-       | Polarity |
|---------------------------|-------|---------|-------|---------|-------|----------|----------|
| K                         | .15   | .15     | .20   | .20     | .25   | .25      | ++-      |
| (AC)                      | +     | +       | +     | +       | +     | +        | produces |
| Polarity (DF)             | +     | +       | +     | +       | +     | +        | no       |
| (GI)                      | +     | -       | +     | -       | +     | -        | signifi- |
| (JL)                      | +     | -       | +     | -       | +     | -        | cant     |
|                           |       |         |       |         |       |          | shifts   |
| Q-x                       | 8.661 | 8.645   | 8.620 | 8.567   | 8.565 | unstable | in       |
| Q-y                       | 8.794 | 8.793   | 8.788 | 8.787   | 8.782 | 8.780    | gamma-   |
| Beta-max (x)              | 47    | 73      | 62    | 37      | 81    | xxx      | tr       |
| Beta-max (y)              | 31    | 32      | 34    | 196     | 38    | 42       |          |
| (dx)-max                  | 5.08  | 7.11    | 7.30  | 8.79    | 11.07 | 15.23    |          |
| Gamma-tr                  | 8.036 | 9.009   | 7.679 | 9.400   | 7.162 | 9.047    |          |
|                           |       |         |       |         |       |          |          |
| $\Delta\sigma_t$ 2-family | ---   | 0.97--- | ---   | 1.73--- |       |          |          |
| $\Delta\sigma_t$ 1-family |       |         | 0.77  | 0.95    |       |          |          |

CONFIGURATION X: 6 lenses: (B-D, G-I, J-L) : 5

| Run No.       | X+    | X+    | X1-   | X1-   | X2-   | X2-   | Polarity    |
|---------------|-------|-------|-------|-------|-------|-------|-------------|
| K             | .20   | .15   | .20   | .15   | .20   | .15   | ++-         |
| (BD)          | +     | +     | -     | -     | +     | +     | gives no    |
| Polarity (GI) | +     | +     | +     | +     | -     | -     | significant |
| (JL)          | +     | +     | +     | +     | +     | +     | gamma-tr    |
|               |       |       |       |       |       |       | shifts      |
| Q-x           | 8.628 | 8.669 | 8.616 | 8.666 | 8.666 | 8.689 |             |
| Q-y           | 8.791 | 8.795 | 8.792 | 8.796 | 8.796 | 8.798 |             |
| Beta-max (x)  | 112   | 67    | 128   | 70    | 57    | 41    |             |
| Beta-max (y)  | 38    | 33    | 37    | 38    | 33    | 26    |             |
| (dx)-max      | 8.54  | 6.97  | 6.73  | 5.62  | 8.39  | 6.77  |             |
| Gamma-tr      | 8.824 | 8.686 | 8.367 | 8.397 | 9.494 | 8.988 |             |
|               |       |       |       |       |       |       |             |
|               |       |       |       |       |       |       |             |
|               |       |       |       |       | 1.04  |       |             |

CONFIGURATION W+: 6 lenses: (B-D, F-H, J-L): 5

| Run No.          | W+    | W+    | W+     | W+     | W+     | W+     | W+     |
|------------------|-------|-------|--------|--------|--------|--------|--------|
| K                | .15   | .20   | .25    | .30    | .35    | .40    | .45    |
| Polarity (BD)    | +     |       |        |        |        |        |        |
| Polarity (FH)    | +     |       |        |        |        |        |        |
| Polarity (JL)    | +     |       |        |        |        |        |        |
|                  |       |       | (ALL)  |        |        |        |        |
| Q-x              | 8.694 | 8.681 | 8.665  | 8.647  | 8.625  | 8.602  | 8.577  |
| Q-y              | 8.798 | 8.796 | 8.793  | 8.790  | 8.787  | 8.783  | 8.779  |
| Beta-max (x)     | 32    | 36    | 40     | 44     | 49     | 54     | 59     |
| Beta-max (y)     | 26    | 27    | 29     | 30     | 31     | 33     | 35     |
| (dx)-max         | 6.93  | 7.78  | 8.98   | 9.99   | 10.82  | 11.48  | 11.99  |
| Gamma-tr         | 9.131 | 9.667 | 10.366 | 11.247 | 12.336 | 13.683 | 15.371 |
| $\Delta\delta_t$ |       | 1.21  | 1.91   | 2.80   | 3.89   | 5.23   | 6.92   |

CONFIGURATION W-: 6 lenses: (B-D, F-H, J-L): 5

| Run No.       | W-    | W-    | W-                  |
|---------------|-------|-------|---------------------|
| K             | .15   | .20   | .25                 |
| Polarity (BD) | +     | +     | +                   |
| Polarity (FH) | -     | -     | -                   |
| Polarity (JL) | +     | +     | +                   |
|               |       |       | as +-+ by symmetry. |
| Q-x           | 8.670 | 8.637 | 8.505               |
| Q-y           | 8.795 | 8.792 | 8.787               |
| Beta-max (x)  | 65    | 98    | 184                 |
| Beta-max (y)  | 33    | 37    | 41                  |
| (dx)-max      | 4.73  | 5.53  | 6.33                |
| Gamma-tr      | 8.457 | 8.445 | 8.414               |

CONFIGURATION E1: 8 lenses: (A4-C6, D4-F6, G4-I6, J4-L6)

| Run No.          | E1/1  | E1/3    | E1/1  | E1/3    | E1/1  | E1/3    |
|------------------|-------|---------|-------|---------|-------|---------|
| K                | .20   | .20     | .25   | .25     | .30   | .30     |
| (AC)             | +     | +       | +     | +       | +     | +       |
| Polarity (DF)    | +     | +       | +     | +       | +     | +       |
| (GI)             | +     | -       | +     | -       | +     | -       |
| (JL)             | +     | -       | +     | -       | +     | -       |
| Q-x              | 8.715 | 8.715   | 8.718 | 8.717   | 8.720 | 8.720   |
| Q-y              | 8.838 | 8.833   | 8.859 | 8.852   | 8.885 | 8.877   |
| Beta-max (x)     | 42    | 41      | 48    | 48      | 56    | 56      |
| Beta-max (y)     | 36    | 30      | 40    | 47      | 45    | 56      |
| (dx) -max        | 4.69  | 7.48    | 5.38  | 9.22    | 6.45  | 11.23   |
| Gamma-tr         | 8.073 | 9.021   | 7.885 | 9.407   | 7.675 | 9.963   |
| $\Delta\delta_t$ | ---   | 0.95--- | ---   | 1.52--- | ---   | 2.29--- |
| (2-family)       |       |         |       |         |       |         |

CONFIGURATION E3: 8 lenses: (A14-C16, D4-F6, G14-I16, J4-L6)

| Run No.          | E3/2  | E3/3    | E3/2  | E3/3    |
|------------------|-------|---------|-------|---------|
| K                | .20   | .20     | .25   | .25     |
| (AC)             | +     | +       | +     | +       |
| Polarity (DF)    | -     | +       | -     | +       |
| (GI)             | +     | -       | +     | -       |
| (JL)             | -     | -       | -     | -       |
| Q-x              | 8.737 | 8.727   | 8.753 | 8.737   |
| Q-y              | 8.841 | 8.833   | 8.864 | 8.854   |
| Beta-max (x)     | 50    | 59      | 63    | 76      |
| Beta-max (y)     | 54    | 61      | 70    | 84      |
| (dx) -max        | 5.08  | 11.10   | 5.77  | 15.29   |
| Gamma-tr         | 8.136 | 9.693   | 7.974 | 10.942  |
| $\Delta\delta_t$ | ---   | 1.55--- | ---   | 2.97--- |
| 2-family         |       |         |       |         |
| $\Delta\delta_t$ |       | 1.24    |       |         |
| 1-family         |       |         |       |         |